Analytical Solutions for Temperature Distribution in Blocks and Application for LCLS II Optics

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Introduction  
The high power X-ray laser produced by LCLS II will be focused using optics that must be cooled to prevent deformation and damage. There are several feasible cooling configurations; performance varies with choice. The goal of this project is to analytically solve for the heat distribution across a block for each cooling configuration. The analytical solutions will allow the most effective cooling system to be chosen for LCLS II optics.  

Keywords: LCLS II, heat distribution, cooling, optics

Problem Setup  
2D Steady State Full Bottom Cooling  
3D Steady State Full Side Cooling  
3D Steady State Bottom Cooling

Results  
2D SS BC  
\[ T(x, z) = T_f + \frac{P_a}{k} \left[ \frac{k}{h} + \frac{z}{h} \right] + \sum_{n=0}^{\infty} \frac{4P_a}{kL \omega_n} \sin \left( \frac{\omega_n h}{2} \right) \frac{1}{\sinh \left( \frac{\omega_n h}{2} \right)} \frac{\cosh \left( \frac{\omega_n h}{2} \right)}{\sinh \left( \frac{\omega_n h}{2} \right)} \cos \left( \omega_n x \right) \]

2D SS SC  
\[ T(x, z) = T_f + \sum_{n=0}^{\infty} \frac{4P_a}{k \alpha_n \omega_n} \sin \left( \frac{\omega_n h}{2} \right) \frac{1}{\sinh \left( \frac{\omega_n h}{2} \right)} \frac{\cosh \left( \frac{\omega_n h}{2} \right)}{\sinh \left( \frac{\omega_n h}{2} \right)} \cos \left( \omega_n x \right) \]

3D SS BC  
\[ T(x, y, z) = T_f + \frac{P_{am}}{k W} \left[ \frac{k}{h} + \frac{z}{h} \right] + \sum_{n=0}^{\infty} \frac{4P_{am}}{k W L^2 \omega_n} \sin \left( \frac{\omega_n h}{2} \right) \frac{1}{\sinh \left( \frac{\omega_n h}{2} \right)} \frac{\cosh \left( \frac{\omega_n h}{2} \right)}{\sinh \left( \frac{\omega_n h}{2} \right)} \cos \left( \omega_n x \right) \]

\[ + \sum_{n=1}^{\infty} \frac{16P_{am}}{k W L \omega_n \beta_n} \sin \left( \frac{\omega_n h}{2} \right) \frac{1}{\sinh \left( \frac{\omega_n h}{2} \right)} \frac{\cosh \left( \omega_n h \right) - \frac{h}{\sinh \left( \omega_n h \right)} \sinh \left( \omega_n h \right) - \frac{h}{\sinh \left( \omega_n h \right)} \cosh \left( \omega_n h \right)}{\sinh \left( \omega_n h \right) + \frac{h}{\sinh \left( \omega_n h \right)} \cosh \left( \omega_n h \right)} \cos \left( \beta_n y \right) \cos \left( \beta_n y \right) \]

MATLAB Results  

Conclusions  
Analytical solutions were obtained for 2D and 3D, steady state and transient, bottom and side cooling. To verify the legitimacy of these solutions, MATLAB was used to graph the results. Bottom cooling solutions were confirmed by extending the size of the beam to the surface of the mirror, making the problem 1D. The next steps in this project are to continue coding and confirming analytical solutions for more complex problems.

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