Evaluation of Flat (X-ray) Mirrors as Alternatives to Prefigured Mirrors

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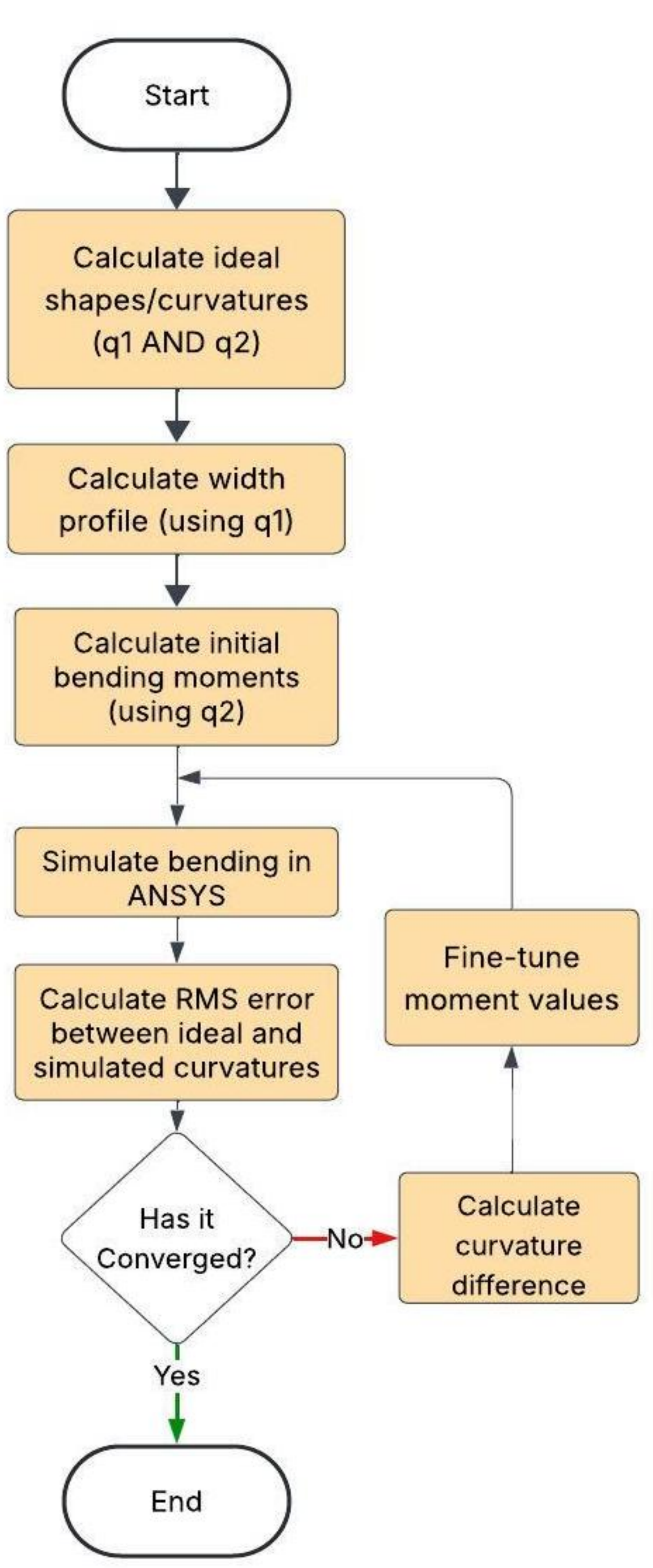
Project Summary

- X-ray beamlines in X-FEL and synchrotron facilities rely on precisely curved mirrors for beam focusing.
- Typically, this is achieved with:
  - Prefigured mirrors, precisely polished to a specific shape, highly accurate but expensive and have extensive lead times.
  - Flat mirrors, which are then bent into shape, significantly cheaper (~90% less than prefigured mirrors).
- This research investigates whether flat, mechanically bent mirrors can replicate the performance of prefigured mirrors when considering multiple focal points.
- Using ANSYS structural simulations and MATLAB-based optimization, we analyze deformation responses to evaluate mirror performance.

Analysis Methods

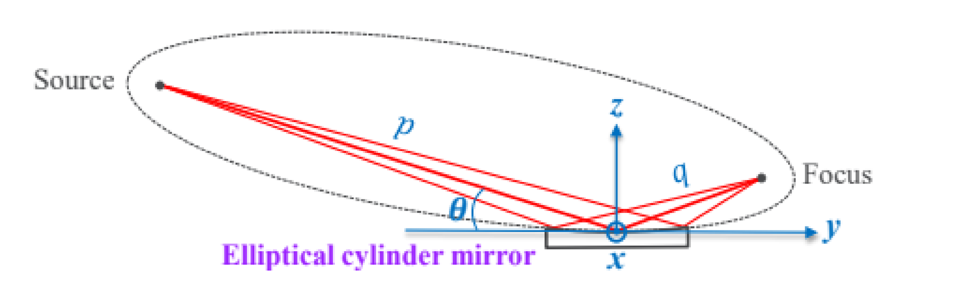
- MATLAB was utilized for data analysis:
  - Ideal ellipse shapes/curvatures.
  - Mirror width profile.
  - Theoretical mirror bending (Euler-Bernouilli)
  - Moment optimization.
- ANSYS simulations were performed to analyze real-world mirror deformation.

Procedure



Analysis and Results

Ideal Mirror Geometry Definition



We define the elliptical shape using standard optical parameters.

- p: source-to-mirror distance.
- q: mirror-to-focus distance.
- θ: grazing angle.

The ellipse shape (centered at and rotated about the point of interest) is given by [2]

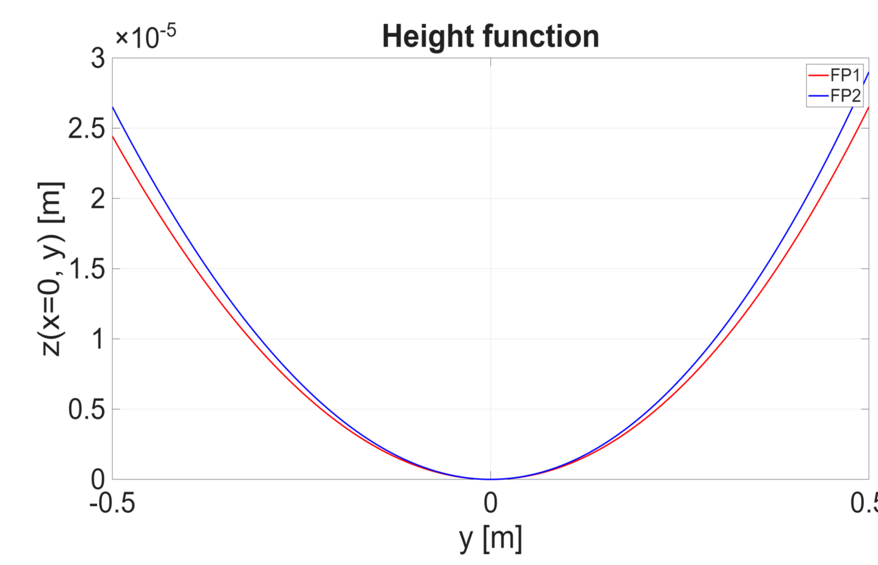
- $z(y) = \frac{(p+q)sin\theta}{4pq+h^2}\left[2pq - hy - 2\sqrt{pq(pq-hy-y^2)}\right]$

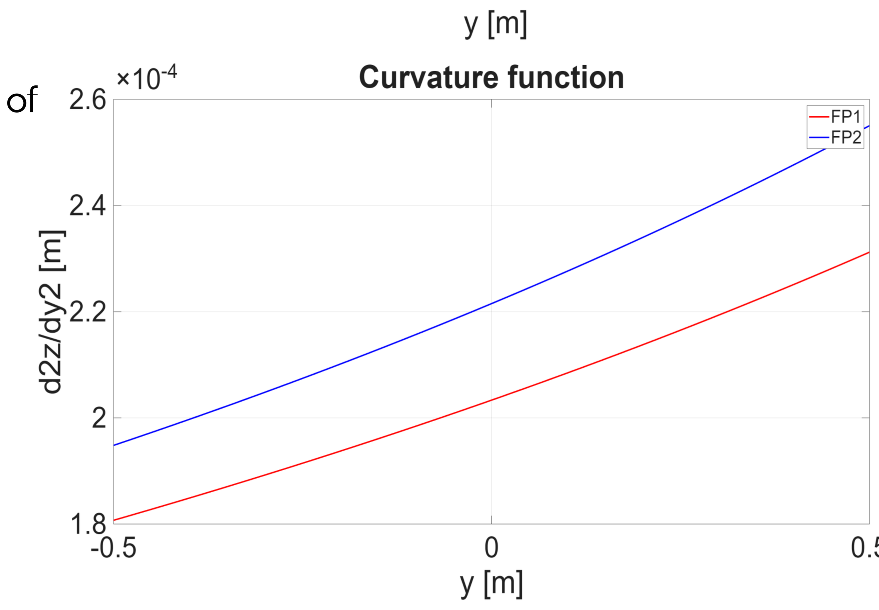
where

- $h = cos\theta(p-q)$

The second derivative (used to approximation curvature) is also provided by [2].

- $\frac{d^2z}{dy^2} = \frac{(p+q)sin\theta}{2pq}\frac{1}{\left(1-\frac{hy+y^2}{pq}\right)^{\frac{3}{2}}}$



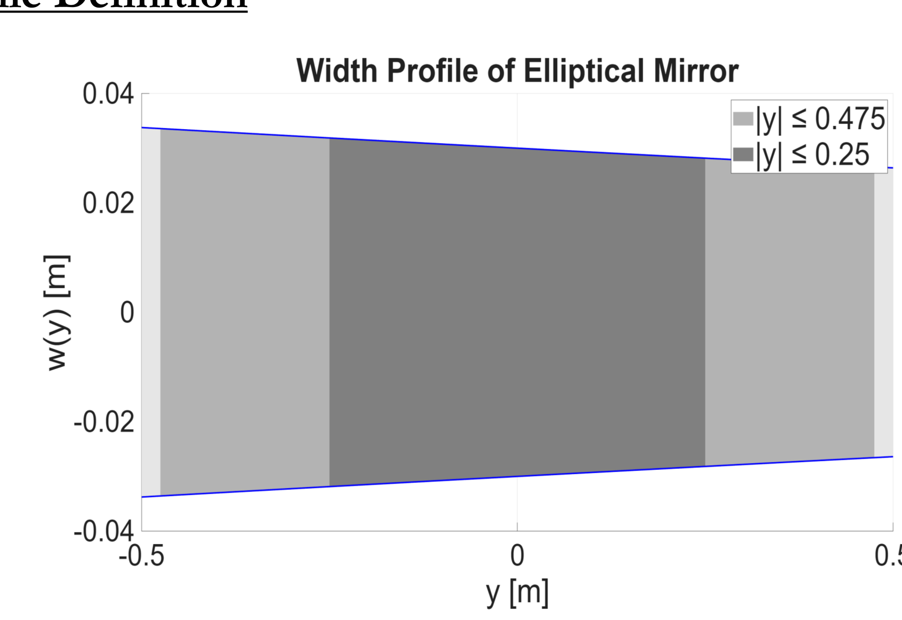


Width Profile Definition

- To achieve best bending performance for the primary focal point, we define a non-linear width profile as:

$$w(y) = \frac{12\,M_0}{Et^3C(y)} = w_0\left[1 - \frac{hy+y^2}{pq}\right]^{\frac{3}{2}} \quad [3]$$

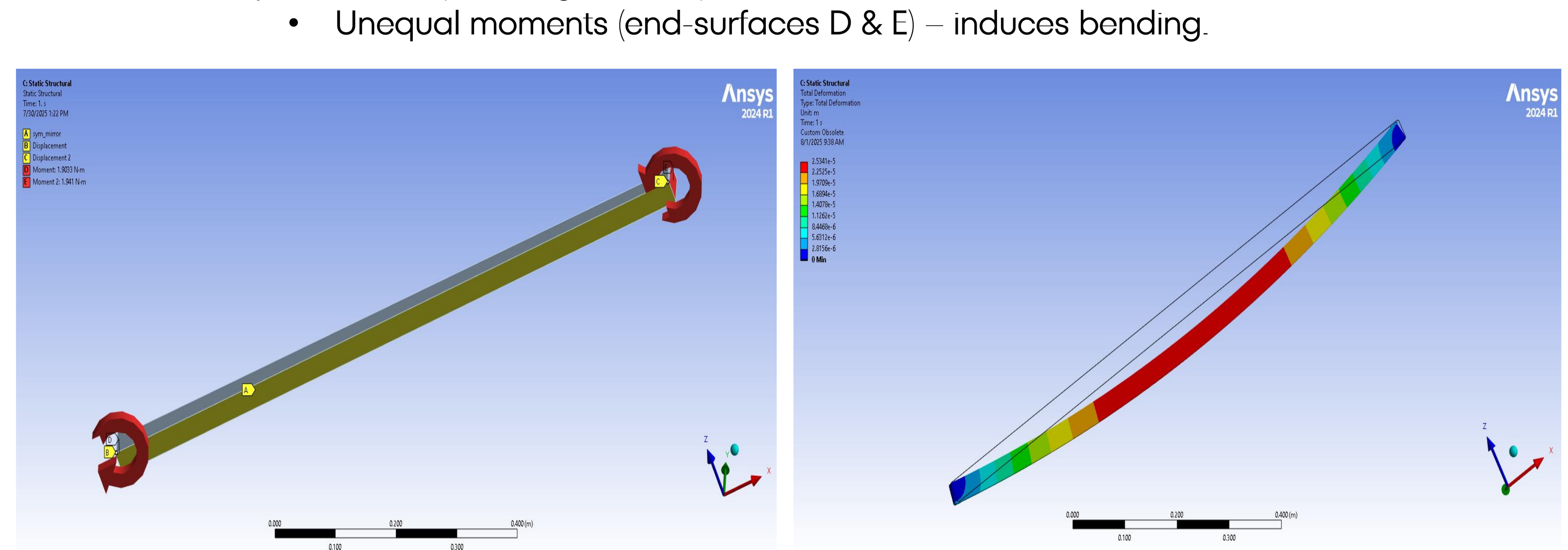
- Here, $C(y)$ is the ideal curvature at Focal Point 1. The non-uniform width ensures that the local stiffness matches the curvature distribution, enabling more precise mirror shaping. This does not consider the mounting geometry present at the mirror ends, which is included in the FEA simulations.



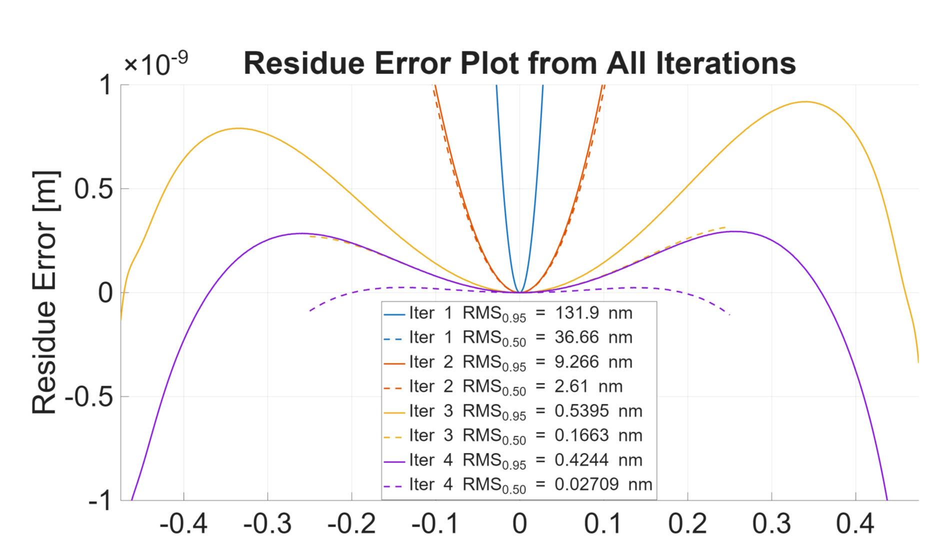
Structural Analysis

To accurately model moment-pair beam bending, the following boundary conditions were applied:

- Mirror Symmetry Plane A – restricts Y-direction movement.
- Displacements (end-edges B & C) – constrain X- and Z-direction movement.
- Unequal moments (end-surfaces D & E) – induces bending.



- After carrying out structural analysis in Ansys, the resulting shape is compared with the ideal shape.
- The difference is called residue, which can be minimized by fine tuning the moments.
- The plot demonstrates the convergence across iterations, highlighting the effectiveness of moment fine-tuning.



Background

We consider the x-ray beam delivery for the MFX beamline at LCLS which requires both focused and defocused beam. The mirror geometry uses:

- Source distance p = 360m.
- Focus distances q_1 = 6 m (FP1), q_2 = 5.5 m (FP2).
- Grazing angle θ = 2.4 mrad.
- Thickness t = 30 mm.
- Center width w_0 = 60 mm.

The mirror material is single-crystal silicon (Si), (100) orientation along the tangential direction, with Young's modulus $E \approx 130$ GPa [1].

Conclusion

- For the MFX test case, we achieve shape errors of
  - RMS_{950mm} = 0.42 nm
  - RMS_{500mm} = 0.027 nm,

  well within the shape error specifications used for manufacturing prefigured mirrors
  - $RMS_{950mm} \leq 1.2$ nm
  - $RMS_{500mm} \leq 0.6$nm
- This confirms that, for small shifts in focal distance (e.g., slight beam defocusing), cost-effective flat mirrors can be used in lieu of expensive prefigured mirrors without degradation in performance.

Future Plans

- **Limits of Flat Mirrors**: Evaluate more extreme cases (with larger differences between focal distances) to determine the limitations of bending flat mirrors.
- **Gravity Sag**: Include the effects of gravity to evaluate the efficacy of this approach for vertically deflecting mirrors.
- **Automate Moment Tuning**: Develop an integrated MATLAB-ANSYS system for automatic iteration and convergence of bending moments to minimize residuals.
- **Experimental Validation**: Physically bend test mirrors and compare with simulated deformation profiles to validated our results.
- **Publish and Open-Source Code**: Finalize documentation and share MATLAB scripts and analysis tools with the optics research community.
- **Stress Analysis**: Carry out stress analysis on mirror and epoxy to ensure mechanical failure does not occur throughout bending.

References

[1] Hopcroft, M. A. et al. (2010). J. Microelectromechanical Syst. 19(2), 229–238.

[2] Goldberg, K. A. (2022). J. Synchrotron Rad. 29, 991.

[3] Zhang, L. Opto-Mechanics Fundamentals BMSCDP Seminar at SLAC.

For a deep dive into my project, scan the qr code or click here

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