

Design and Optimization of Herriot Cells for Non-Linear Optics

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LCLS II Upgrade

Background

In September of 2023, LCLS II, the long awaited upgrade to LCLS, produced its first x-rays. This upgrade, among other feats, increased the x-ray pulse repetition rate by orders of magnitude. Pump-probe experiments require a matching upgrade in optical and infrared laser systems. Consequently, there is now a pressing need to increase pulsed laser systems average power output and frequency conversion efficiency at LCLS.

Objective

We suggest a multi-pass cell for both self-phase modulation and increased efficiency in parametric down conversion, which has seen great promise in recent years. To engineer a multi-pass cell, we require simulations to understand beam propagation and non-linear interactions within the cell.

Gaussian Beam Propagation

Ray Transfer Matrix

In geometric optics, an ABCD matrix is used to determine the trajectory of a beam with the following relation.

$$\begin{pmatrix} x_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix}$$

For our system, we can treat a spherical mirror as a thin lens and so our ABCD matrix for an N pass cell becomes

$$M^N = \left\{ \begin{pmatrix} 1 & d/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & d/2 \\ 0 & 1 \end{pmatrix} \right\}^N$$

Gaussian Beams

The fundamental transverse mode of a laser resonator (TEM₀₀) is a solution to the Helmholtz Equation constrained to having spherical wavefronts.

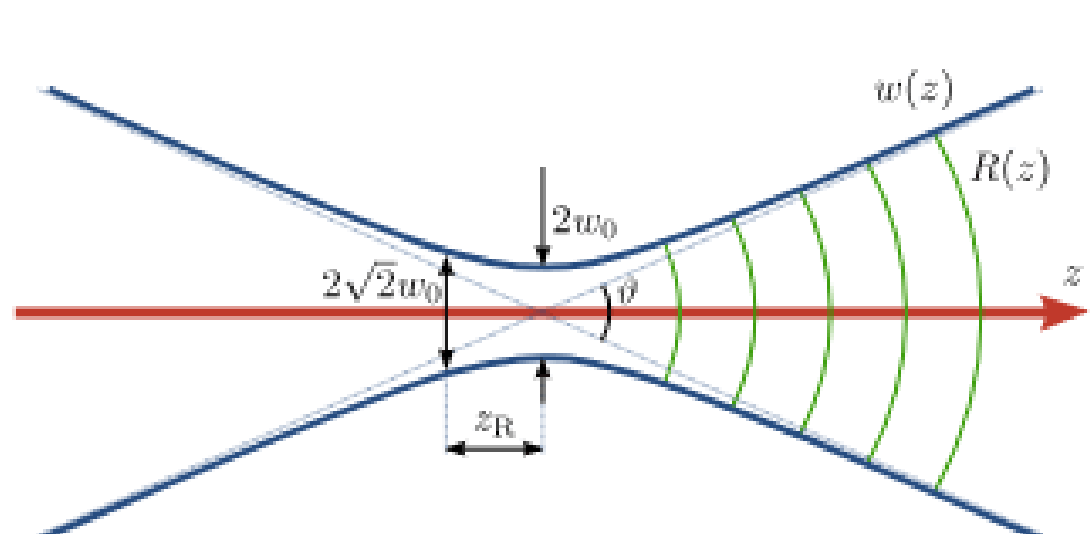
$$U(r, z) = \frac{w_0}{w(z)} \exp\left\{-i(kz - \phi) - r^2 \left(\frac{1}{w(z)^2} + \frac{ik}{2R(z)} \right)\right\}$$

The complex beam parameter $q(z)$ describes the transverse evolution of the beam given the following relation.

$$\frac{1}{q(z)} = \frac{1}{R(z)} + \frac{i\lambda}{\pi w(z)^2}$$

In tandem with the ABCD matrix, we can solve for the beam waist $w(z)$ and the wavefront's radius of curvature $R(z)$ inside our cell computationally, using

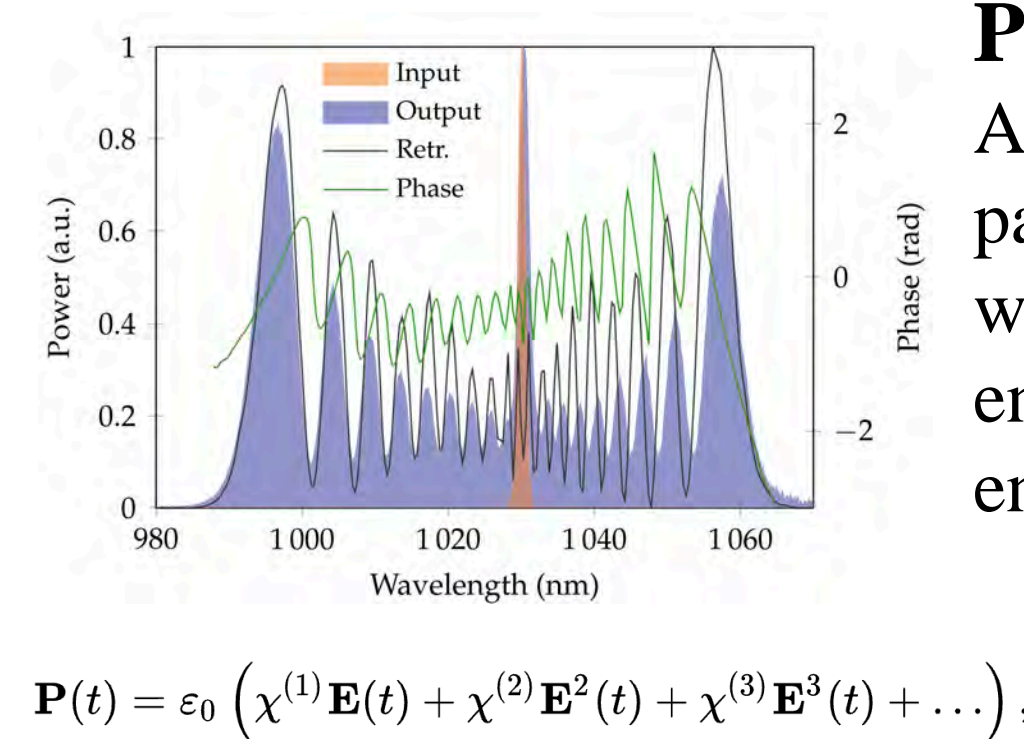
$$BD + q^2 AC = 0 \quad w_1 = \sqrt{w_0^2 (A^2 + \frac{B^2}{q^2}) / (AD - BC)}$$



Non-Linear Interactions

Self-Phase Modulation

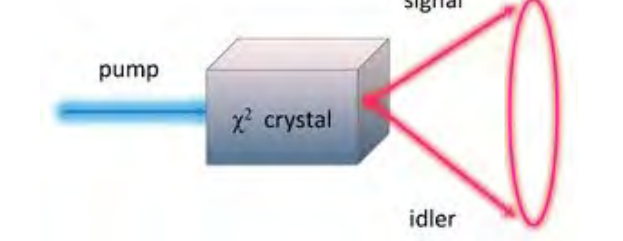
For pulse compression, the beam will propagate through a gaseous medium which will induce a varying refractive index. The resulting phase shift will broaden the beam's spectrum allowing for shorter pulses in time.



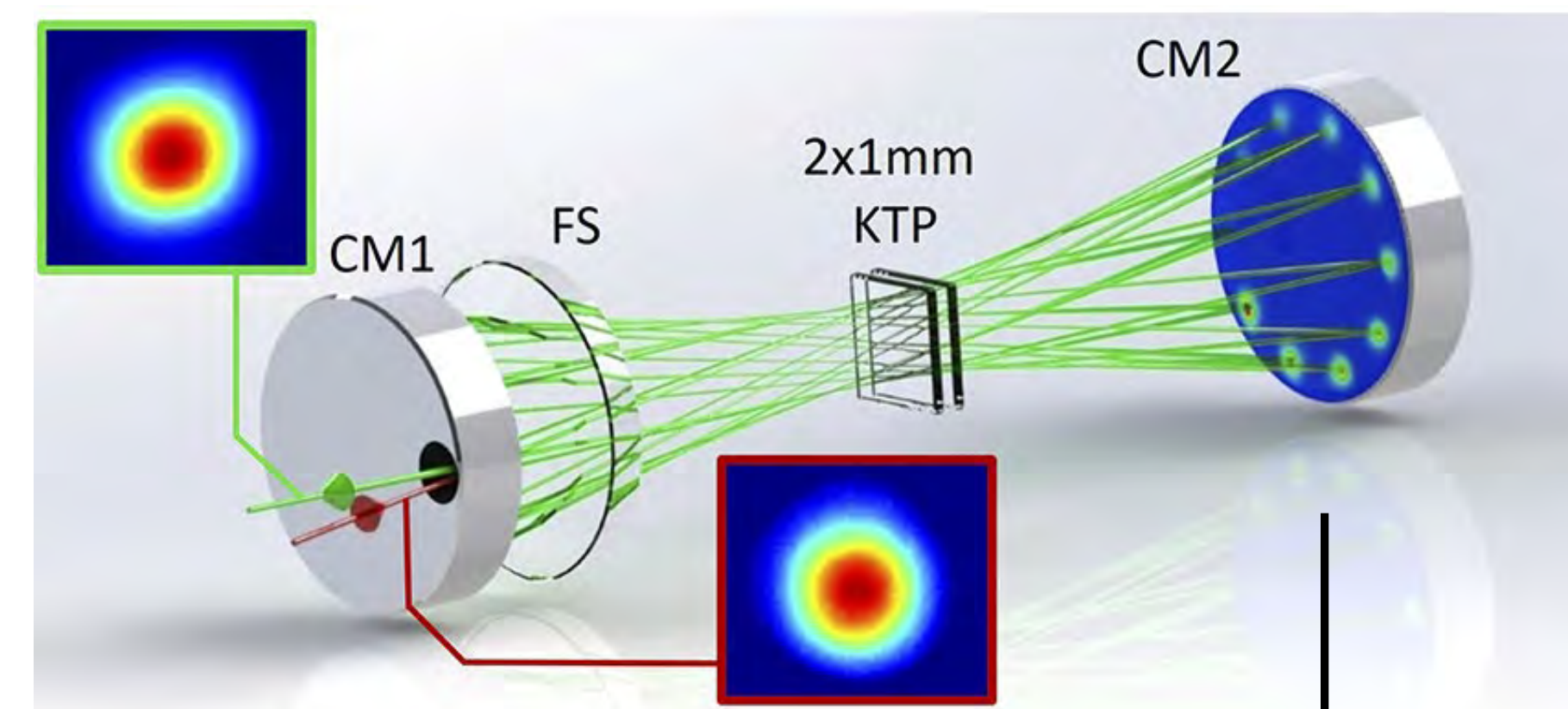
$$P(t) = \epsilon_0 \left(\chi^{(1)} \mathbf{E}(t) + \chi^{(2)} \mathbf{E}^2(t) + \chi^{(3)} \mathbf{E}^3(t) + \dots \right)$$

Parametric Down Conversion

A higher frequency pump beam will pass through the cell interacting with a crystal that converts higher energy photons into a pair of lower energy photons (signal and idler).



Stable Mode Simulations



Cell Geometry

With two spherical mirrors the beams propagation forms an ellipse governed by cell and input beam parameters.

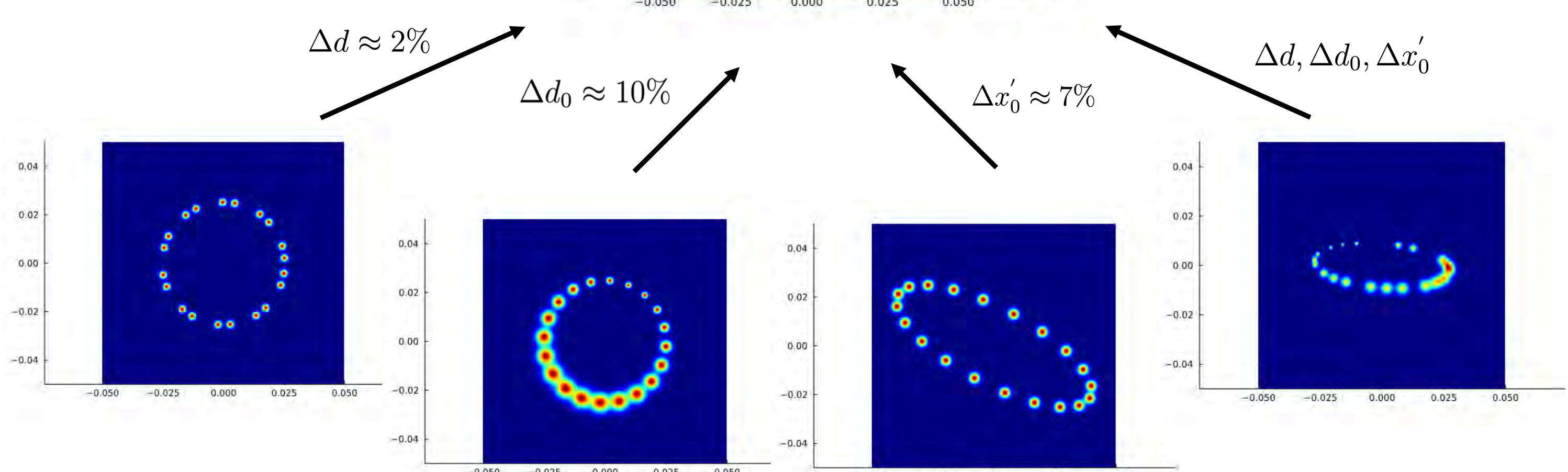
$$x_n = x_0 \cos n\theta + \sqrt{\frac{d}{4f-d}} (x_0 + 2fx_0') \sin n\theta$$

$$\cos \theta = 1 - (d/2f)$$

Stable Mode Conditions

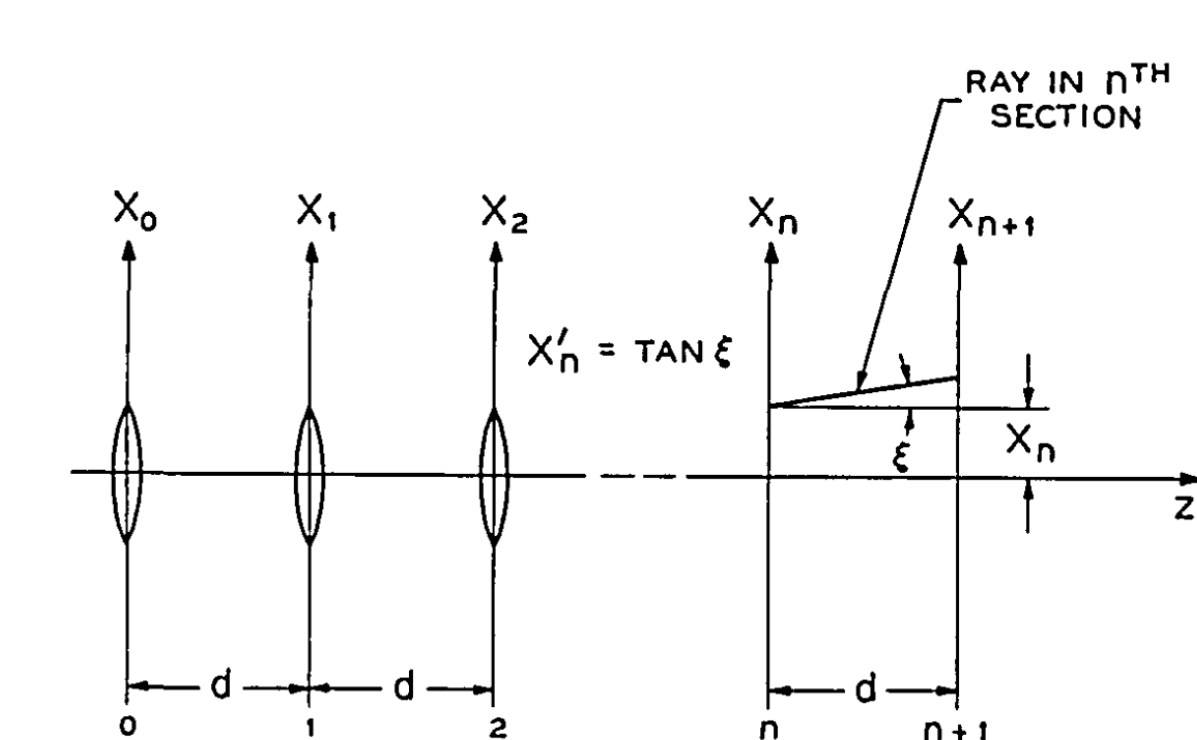
To maintain a stable circular pattern on the mirror, the parameters must meet these conditions.

$$d = 2f(1 + \cos(\frac{\pi}{N})) \quad x_0' = -\frac{2x_0}{d}$$

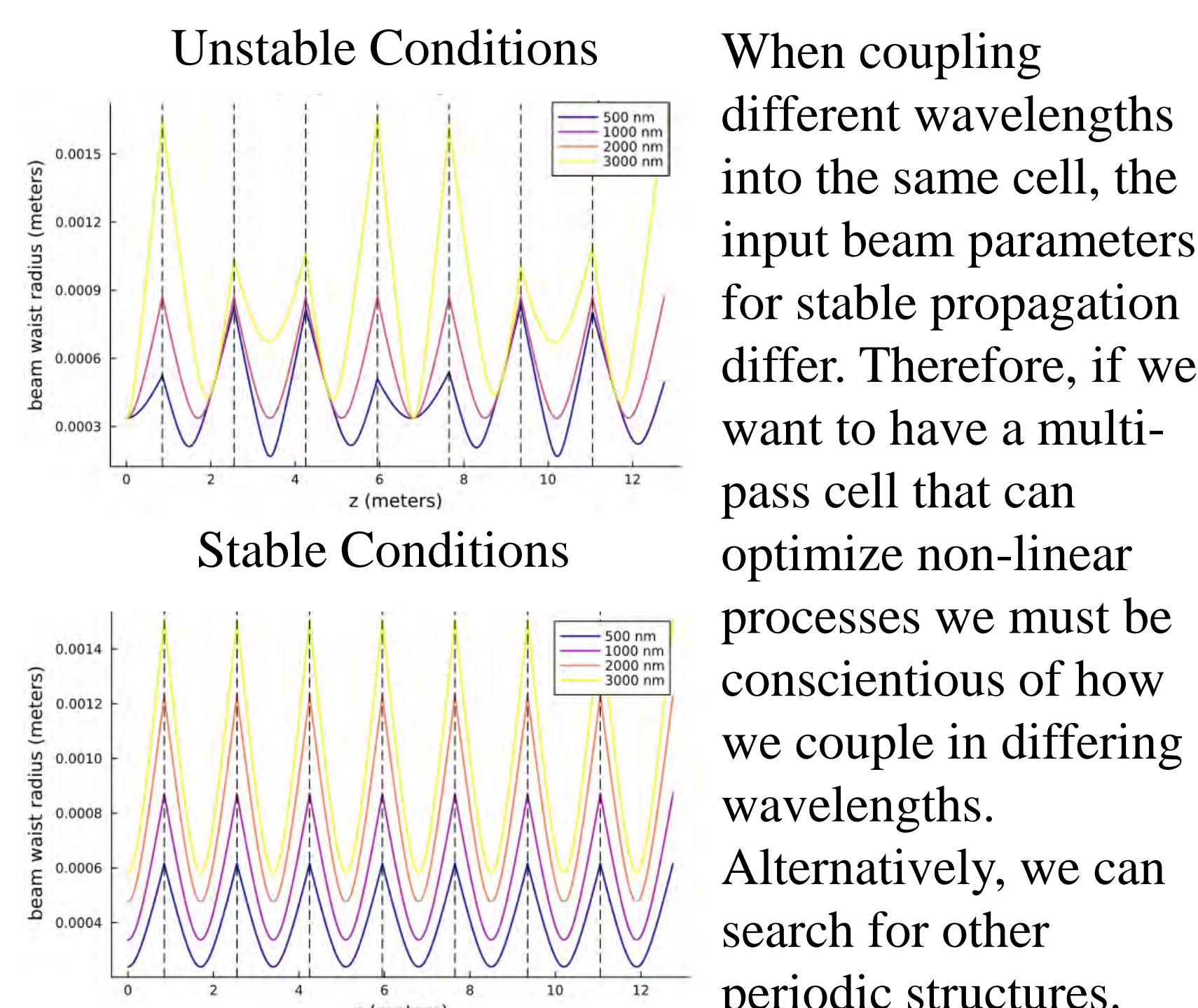


Eigenmodes For Periodic Lenses

There exist modes where the complex beam factor q is periodic inside a system of equidistant lenses for given in coupling beam parameters. Because spherical mirrors have the same effect on gaussian beams as a thin lens, we can use these results for our multi-pass cell.



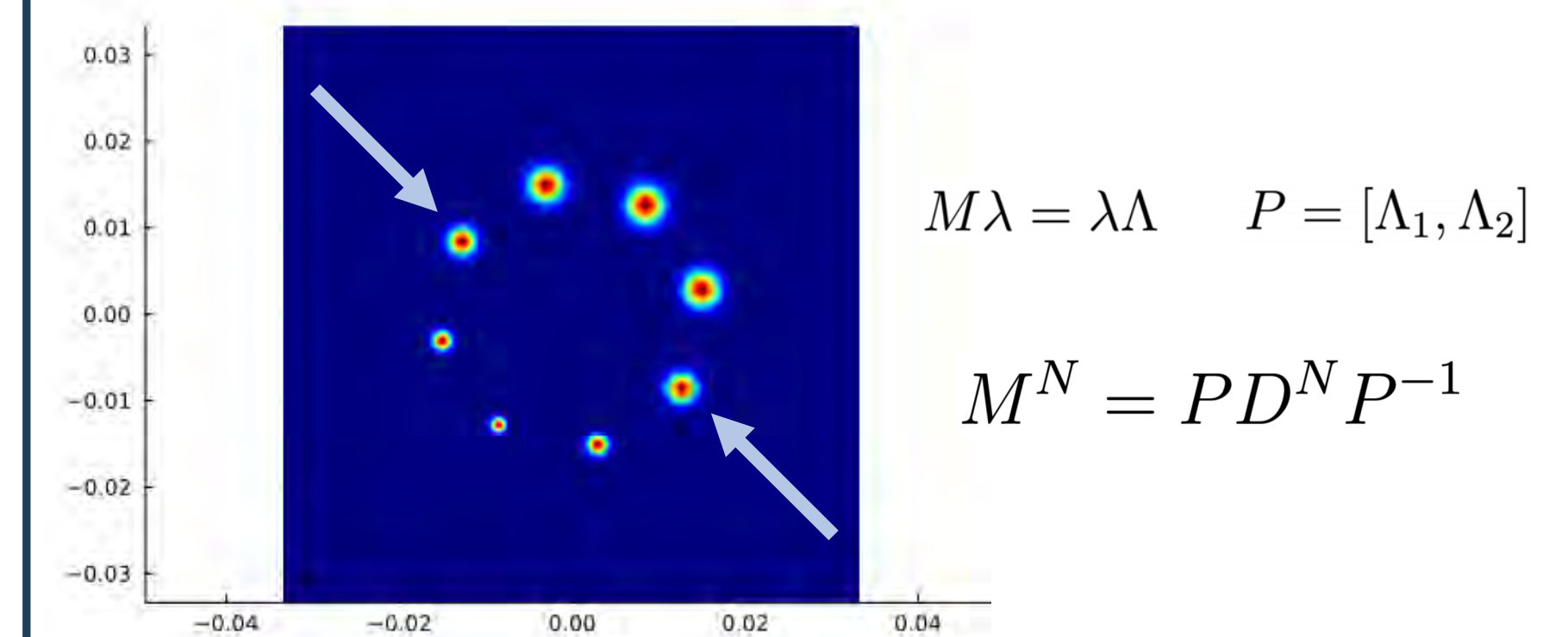
Multi-Color Beam Propagation



Imaging Conditions

Imaging Conditions

When $B = 0$ in the ABCD matrix, we reach an imaging condition. If stable mode conditions are met in the cell, we can image an identical beam at pass number $N/2$ and N .



This will give us another place to couple out a beam with the same size as the input. We can efficiently compute the conditions for imaging in Mathematica by diagonalizing the transfer matrix, M .

Self-Focusing Correction

Change to Complex Beam Factor

Self-focusing occurs when a non-uniform (gaussian) beam of light alters the intensity dependent index of refraction in a non-linear medium. It is suggested that for gaussian beams, the self focusing affects the complex beam factor as follows.

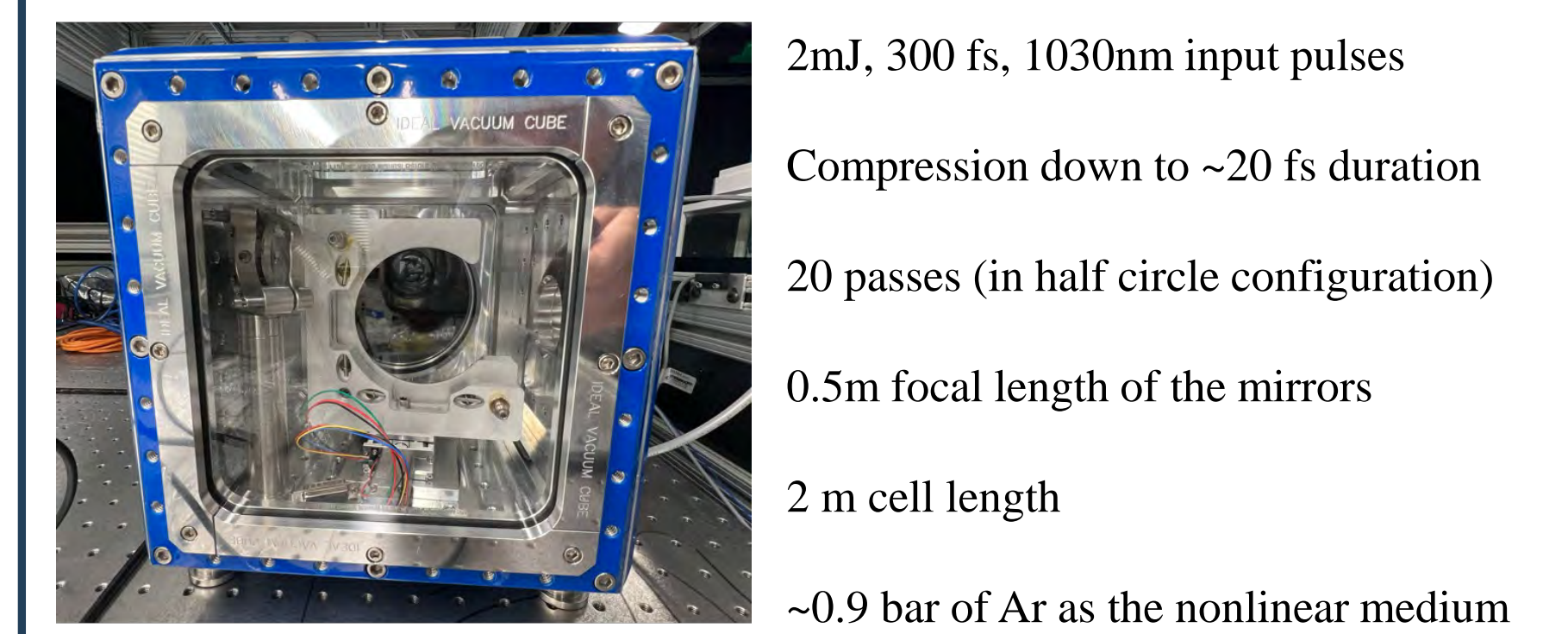
$$\frac{1}{q(z)} = \frac{1}{R(z)} + \frac{i\sigma\lambda}{\pi w(z)^2} \quad \sigma = 1 - P/P_{crit}$$

This will alter our alignment procedure, as the focusing conditions for a high intensity beam will be different than for a low intensity alignment beam. In the future, we will do further simulations to better understand how to align a low intensity beam, so that the high intensity pulsed beam is focused properly.



MPC Design for LCLS II

Viewport



Sideview



We can use the viewport to image beam propagation inside the cell.

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CITATIONS

- "Multipass spectral broadening of 18 mJ pulses compressible from 1.3 ps to 41 fs," *Opt. Lett.* 43, 5877-5880 (2018)
- "Nonlinear beam matching to gas-filled multipass cells," *OSA Continuum* 4, 732-738 (2021)
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- "Temporal quality of post-compressed pulses at large compression factors," *J. Opt. Soc. Am. B* 39, 1694-1702 (2022)